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Modified Adomian Decomposition Method for the Solution of Integro-Differential Equations

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1 Introduction

The integro-differential equation is a type of mathematical expression that includes the derivative and integral transforms of a required function. These equations are common in processes where the value of a quantity of interest, which is a function at each point, is clearly determined by its proximity to the points described by the differential equation, but it also depends on the function's distribution over the domain.

Numerous fields rely on integralo-differential equations, including electrostatics, engineering, mechanics, astronomy, physics, and potential theory. Because of how difficult these equations are to solve analytically, numerical methods are often used. The solution of integro-differential equations has been approached using several numerical approaches in recent years. Some of these methods are the multistep method and the spectral collocation method. Check out Guslu and Sezer [3], Cao and Wang [4], and Bhrawy et. al [5] for a few writers who have devoted a lot of time to integro-differential equations. To resolve singular ordinary differential equations, Ine and Evans [6] used the Adomian Decomposition Method (ADM). The novel modified decomposition approach was used by Kumar and Singh [7]. In order to solve high-order nonlinear Volterra-Fredholm integro-differential equations with separable kernels, Behiry [8] used the differential transformation approach. Using the methods of Modified Homotopy Plotting, Variation Iteration, Homotopy Analysis, and Modified Adomian Decomposition, Behzadi [9] was able to solve a nonlinear Volterra-Fredholm integro differential equation in two dimensions.

1.1 General problem considered

In this paper, the basic ideas of the research work done by Adomian and Wazwaz were modifiedand applied to high-order non-linear Volterra-Fredholm integro-differential equation of the form:

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2 Adomain Decomposition Method

The Adomian decomposition method is a well-known systematic method for practical solution oflinear or nonlinear and deterministic or stochastic operator equations, including ordinary differential equations, partial differential equations, integral equations, integro-differential equations, to mention

but few. The Adomian decomposition method is a powerful techniques, which provides efficient algorithms for analytic approximate solutions and numerical simulations for real-world applications in the applied sciences and engineering. It permits us to solve both nonlinear initial value problems (IVPs) and boundary value problems (BVPs) without unphysical restrictive assumptions such as required by linearization, perturbation, guessing the initial term or a set of basis functions, and so forth. Furthermore the Adomian decomposition method does not require the use of Green'sfunctions, which would complicate such analytic calculations since Green's functions are not easily determined in most cases. The accuracy of the analytic approximate solution obtained can be verified by direct substitution.

Table of Results

Table 1. Approximate solutions of *ADM* and *MADM* methods

x	Exact Solution	ADM SOLUTION	MADM SOLUTION
0	1	1	1
0.1	1.105170918	1.106584571	1.105099699
0.2	1.221402758	1.232265285	1.226644901
0.3	1.349858808	1.38138354	1.370958679
0.4	1.491824698	1.555642294	1.544556869
0.5	1.648721271	1.752621335	1.752602717
0.6	1.822118800	1.996767498	1.963593484
0.7	2.013752707	2.271661348	2.170330117
0.8	2.225540928	2.559078232	2.340455296
0.9	2.459603111	2.420727359	2.818867744
1.0	2.718281828	2.3273762	2.974691362

Table 2. Absolute error of example 1

x	Absolute Error for MADM	Absolute Error for ADM
0	0	0
0.1	1.4137×10^{-3}	7.1219×10^{-5}
0.2	1.0863×10^{-2}	5.2421×10^{-3}
0.3	3.1525×10^{-2}	2.1100×10^{-2}
0.4	6.3818×10^{-2}	5.2732×10^{-2}
0.5	1.0390×10^{-1}	1.0388×10^{-1}
0.6	1.7465×10^{-1}	1.1415×10^{-1}
0.7	2.5791×10^{-1}	1.5658×10^{-1}
0.8	3.3354×10^{-1}	1.1150×10^{-1}
0.9	3.8876×10^{-1}	3.5926×10^{-1}
1.0	3.9091×10^{-1}	2.5641×10^{-1}

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Table 3. Solution OF example 2

x	Exact solution	ADM SOLUTION	MADM SOLUTION
0	1	1	1
0.1	0.9999984769	0.9949635001	0.995035117
0.2	0.9999939077	0.980428129	0.9811427381
0.3	0.9999862922	0.9611371238	0.958049452
0.4	0.9999756307	0.9486963588	0.933130990
0.5	0.9999619231	0.9702455385	0.916434756
0.6	0.9999451694	0.927343582	1.078590085
0.7	0.9999253697	1.368195179	0.997532716
0.8	0.9999025240	1.997170465	1.175560711
0.9	0.9998766325	3.222169017	1.532907813
1.0	0.9998476952	2.170869744	5.445197827

Table 4. Absolute error for example 2

x	ADM ABSOLUTE ERROR	MADM ABSOLUTE ERROR
0	0	0
0.1	5.03498×10^{-3}	4.9634×10^{-3}
0.2	1.9566×10^{-2}	1.8851×10^{-2}
0.3	3.8849×10^{-2}	4.1937×10^{-2}
0.4	5.1279×10^{-2}	6.6845×10^{-2}
0.5	2.9716×10^{-2}	8.3527×10^{-2}
0.6	7.2602×10^{-2}	7.8645×10^{-3}
0.7	2.3927×10^{-3}	3.6827×10^{-1}
0.8	9.9727×10^{-1}	1.7566×10^{-1}
0.9	2.2223×10^{1}	5.3303×10^{-1}
1.0	1.1710×10^{1}	4.4454×10^{1}

5 Conclusion

In this paper, Adomian decomposition method and Modified Adomian decomposition method wasused to solve linear and non-linear Volterra-Fredholm integro-differential equations. From the tables of results, we observed that Modified Adomian decomposition method is more efficient, reliable and less computational in terms of cost.

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